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Cyclotron resonance of polarons in ternary mixed crystals

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Abstract. Rayleigh-Schrödinger perturbation theory and an improved Wigner-Brillouin perturbation theory has been used to study the cyclotron resonance of the polarons in ternary mixed crystals in the zero-temperature limit. The interaction between an electron and two branches of longitudinal optical phonon modes is taken into account in the framework of the random-element-isodisplacement model. The numerical results for several ternary mixed crystals show that the polaronic cyclotron energy and mass split successively twice related to the higher and lower branches of longitudinal optical phonon modes of ternary mixed crystals. A non-linear dependence of the polaronic cyclotron energy and mass on the composition x is found.

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1 Introduction

Ternary mixed crystal (TMC) materials are of great importance for the modern electronics. Their properties may be varied over a wide range by changing the composition of the mixed crystal. Some of the most important physical properties of the systems related to their lattice vibration and electron-phonon (e-p) interaction. Quite a lot of experimental and theoretical studies on the lattice dynamics and the e-p coupling in TMCs have been done [1–10]. The properties of the lattice vibration in a TMC are more complicated than those in a binary crystal because of their alloy nature. So-called one-mode, two mode and other mixture characteristics of optical phonon modes were observed and analyzed [1–3]. Some authors have used the random-element-isodisplacement (REI) model and its modified model called the modified random-element-isodisplacement (MREI) model [5,6] to discuss successfully the mode behavior of the lattice vibration in TMCs. One of the authors and his collaborator have improved the previous work and correctly derived the canonical modes of the optical vibration and a new Fröhlich-like e-p coupling Hamiltonian in a recent work [10]. The calculated mode behavior is in agreement with the previous experimental observations [1].

Cyclotron resonance (CR) measurement is an important method to investigate both the characteristics of electrons and phonons in polar materials. In recent years much attention has been paid to polaron problems in an external magnetic field [11–14]. The magnetopolarons in binary polar crystals have been widely investigated by previous authors. The Rayleigh-Schrödinger perturbation theory (RSPT), Wigner-Brillouin perturbation theory (WBPT) and its improvement (IWBPT) are usually applied to treat the effect of the e-p interaction on Landau levels [11]. Even a few works have preliminarily studied the polaron problems in TMCs, the CR of polarons has been rarely investigated theoretically to our knowledge.

In the present work we study the CR of polarons in TMCs. In Section 2 we write down the Hamiltonian of an e-p system in a TMC with an external magnetic field, on the basis of the new e-p interaction Hamiltonian [10]. The two-mode characteristic of the lattice vibration in TMCs is considered in the formulation of the e-p interaction. By using the RSPT and IWBPT similar to those used by Lindemann *et al.* [11] to investigate the CR of polarons in binary polar crystals the energy levels of polarons in a TMC are obtained in Section 3. The CR energy and effective mass of the polaron in a TMC are given and analyzed in Section 4. The numerical computations for the polaronic CR energies and masses in several TMCs are performed and the results are discussed in Section 5.

2 Hamiltonian

We consider an electron in a TMC $A_xB_{1-x}C$. A uniform magnetic field $\mathbf{B}$ along the z direction is applied. Here x is the composition of A-ion in the TMC. As is well known, there are two branches of longitudinal optical (LO) phonon modes coupling with the electron in the system.

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The Hamiltonian of the e-p system then can be written as

$$H = H_e + H_{ph} + H_{e-LO}. \tag{1}$$

For the sake of easiness, we supposed that the band mass of the electron is isotropic.

The first term in equation (1) is the Hamiltonian of an electron in a magnetic field described by

$$H_e = \frac{1}{2m_b}\left(p_x - eBy\right)^2 + \frac{1}{2m_b}\left(p_y^2 + p_z^2\right), \tag{2}$$

where p_x, p_y and p_z are respectively the x, y and z components of the momentum $\mathbf{p}$ of the electron. m_b is the electron band-mass and described usually by the following linear interpolation model [4,7]

$$m_b = xm_{bA} + (1-x)m_{bB}. \tag{3}$$

m_{bA} and m_{bB} refer to the band masses of the electron for the end-member binary crystals AC and BC respectively. Furthermore, the Laudau gauge is employed here, *i.e.*, the vector potential of the magnetic field is taken to be $\mathbf{A} = (-By, 0, 0)$. The second term in equation (1) is the free-phonon-field Hamiltonian

$$H_{ph} = \sum_{j\mathbf{k}} \hbar\omega_{jL} a^\dagger_{j\mathbf{k}} a_{j\mathbf{k}}, \tag{4}$$

where $a^+_{j\mathbf{k}}$ and $a_{j\mathbf{k}}$ are respectively the creation and annihilation operators of the LO-phonon with frequency ω_{jL} and wave-vector $\mathbf{k}$. It is to be noted that there are two branches of LO-phonon modes labeled by the subscript j, where $j = +$ and $-$ stand for respectively the higher- and lower-frequency branches of the LO-phonon modes in the TMC.

The last term in equation (1) describes the interaction between the electron and two branches of LO-phonon modes given by [10]

$$H_{e-Lo} = \sum_{j\mathbf{k}} \left[G^*_j(k)\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}} a^\dagger_{j\mathbf{k}} + \text{h.c.}\right]. \tag{5}$$

The e-p coupling function in equation (5) is written as

$$G^*_j(k) = \frac{g^*_j}{kV^{1/2}}, \tag{6}$$

where

$$g^*_+ = \frac{\mathrm{i}e}{\varepsilon_0 + b_{33}}\left(\frac{\hbar}{2\omega_{+L}}\right)^{1/2}\left(\frac{1}{T_{11} + 2B_1T_{12} + B_1^2T_{22}}\right)^{1/2} \times \left(b_{31} + B_1b_{32}\right), \tag{7a}$$

$$g^*_- = \frac{\mathrm{i}e}{\varepsilon_0 + b_{33}}\left(\frac{\hbar}{2\omega_{-L}}\right)^{1/2}\left(\frac{1}{T_{11} + 2B_2T_{12} + B_2^2T_{22}}\right)^{1/2} \times \left(b_{31} + B_2b_{32}\right). \tag{7b}$$

In (7a) and (7b), e and ε_0 are the electronic charge and the vacuum dielectric constant respectively. The parameters B_1 and B_2 are determined as

$$B_1 = -\frac{T_{12}b'_{21} + T_{11}\left(b'_{11} + \omega^2_{+L}\right)}{T_{11}b'_{12} + T_{12}\left(b'_{22} + \omega^2_{+L}\right)}, \tag{8a}$$

$$B_2 = -\frac{T_{12}b'_{21} + T_{11}\left(b'_{11} + \omega^2_{-L}\right)}{T_{11}b'_{12} + T_{12}\left(b'_{22} + \omega^2_{-L}\right)}. \tag{8b}$$

Here $T_{11} - T_{22}$ are the elements of the kinetic energy matrix and given by

$$T_{11} = \frac{M_Ax\left[M_C + M_B(1-x)\right]}{\mu_A M}, \tag{9a}$$

$$T_{12} = T_{21} = -\frac{M_AM_Bx(1-x)}{\sqrt{\mu_A\mu_B}M}, \tag{9b}$$

$$T_{22} = \frac{M_B(1-x)\left(M_C + M_Ax\right)}{\mu_B M}, \tag{9c}$$

where M_A, M_B and M_C are the masses of ions A, B and C respectively. M is the effective unit-cell mass of the TMC written as

$$M = M_C + xM_A + (1-x)M_B. \tag{10a}$$

μ_A and μ_B are, respectively, the reduced masses of the ion-pairs in binary crystals AC and BC given by

$$\mu_A = \frac{M_CM_A}{M_C + M_A} \quad \text{and} \quad \mu_B = \frac{M_CM_B}{M_C + M_B}. \tag{10b}$$

The frequencies of the two branches of LO-phonon modes ω_{+L} and ω_{-L} in the above equations are determined by

$$\omega^2_{\pm L} = \frac{1}{2}\left\{-\left(b'_{11} + b'_{22}\right)^2 \pm \left[\left(b'_{11} - b'_{22}\right)^2 + 4b'_{12}b'_{21}\right]^{1/2}\right\}. \tag{11}$$

The dynamical coefficients b'_{11} to b'_{22} and b_{11} to b_{33} in the above equations are given in reference [8] and omitted here for short.

For the convenience of calculation, we introduce the ratio of the unperturbed cyclotron frequency $\omega_c = eB/m_b$ to the frequency of higher-branch LO-phonons ω_{+L}

$$\lambda^2 = \omega_c/\omega_{+L} = Be/m_b\omega_{+L} \tag{12}$$

to describe the magnetic field strength and use the so-called "polaron units" related to the same phonon modes. All lengths are in units of the polaron radius defined by $r_p = \left(\hbar/2m_b\omega_{+L}\right)^{1/2}$ and energies are in units of the LO-phonon energy of the higher-frequency branch. Thus equation (2) is rewritten as following dimensionless form

$$H_e = \left(p_x - \frac{1}{2}\lambda^2 y\right)^2 + p_y^2 + p_z^2. \tag{13}$$

The corresponding changes are also applied to equations (4, 5).

3 Perturbation theory

To investigate the CR of polarons we shall calculate the energies of $n = 0$ and $n = 1$ Landau levels respectively. For this purpose we employ a perturbation theory similar to that was used to study the CR of polarons in binary crystals [11–13].

Firstly, let us consider the $n = 0$ states by using the second-order RSPT.

The Hamiltonian (1) can be written as a summation of two terms

$$H = H_0 + H_1. \tag{14}$$

The first term without involving the e-p interaction

$$H_0 = H_e + H_{ph} \tag{14a}$$

is solvable and considered as the unperturbed Hamiltonian. The second term describes the e-p coupling energy and can be treated as a perturbation

$$H_1 = H_{e-LO}. \tag{14b}$$

The unperturbed eigenvectors of equation (14) are chosen as the eigenstates of the free Hamiltonian H_0

$$\left|n, q_z\right\rangle\left|\mathbf{k}\right\rangle. \tag{15}$$

Here $|n, q_z\rangle$ is the eigenfunction of the electron situated in the nth Landau level with the wave-vector q_z in the z-direction and

$$\left|\mathbf{k}\right\rangle = \prod_j a^{\dagger}_{j\mathbf{k}_1} a^{\dagger}_{j\mathbf{k}_2} ... a^{\dagger}_{j\mathbf{k}_N} \left|0_j\right\rangle \tag{15a}$$

is the phonon state-vector. $\left|0_j\right\rangle$ stands for the non-phonon state of jth branch of LO-phonons and the vacuum state of the phonon field is described by

$$\left|0_j\right\rangle = \prod_j \left|0_j\right\rangle. \tag{15b}$$

The unperturbed energy levels related to the nth Landau level and zero phonon state are easily obtained as

$$E_n^{(0)} = \left\langle 0\right| \left\langle n, q_z\right| H_0 \left|n, q_z\right\rangle \left|0\right\rangle = \left(n + \frac{1}{2}\right)\lambda^2 + q_z^2. \tag{16}$$

The energies of $n = 0$ states are then given by

$$E_0^{(0)} = \frac{1}{2}\lambda^2 + q_z^2. \tag{17}$$

The first-order perturbation correction to the $n = 0$ states is obviously zero and the second-order perturbation correction can be calculated by

$$\Delta E_0^{(2)} = \sum_{n=0}^{\infty} \sum_{j\mathbf{k}} \frac{\left|\left\langle \mathbf{k}\right| \left\langle n, q_z\right| H_1 \left|0, q_z\right\rangle \left|0\right\rangle\right|^2}{D_{0n}^{RSPT}}, \tag{18}$$

where

$$D_{0n}^{RSPT} = E_0^{(0)} - \left(E_n^{(0)} + \delta_j\right). \tag{19}$$

and

$$\delta_j = \hbar\omega_{jL}/\hbar\omega_{+L}. \tag{20}$$

δ_j is the dimensionless phonon energy of the jth branch of LO phonons.

Since our attention is focused on the CR of polarons, q_z can be put to be zero without loss of the generality. Substituting (16, 17) and (19) into (18), the second-order correlation to the $n = 0$ states is written as

$$\Delta E_0^{(2)} = -\sum_{j\mathbf{k}} \left|G_j(k)\right|^2 \exp\left[-\left(k_\perp/\lambda\right)^2\right] \times \sum_{n=0}^{\infty} \frac{\left(k_\perp/\lambda\right)^{2n}}{n!\left(n\lambda^2 + \delta_j + k_z^2\right)}, \tag{21}$$

where $k_\perp^2 = k_x^2 + k_y^2$.

After transforming the last factor in equation (21) to be an integral

$$\frac{1}{n\lambda^2 + \delta_j + k_z^2} = \int_0^{\infty} \mathrm{d}t \exp\left[-\left(n\lambda^2 + \delta_j + k_z^2\right)t\right], \tag{22}$$

one can obtain

$$\Delta E_0^{(2)} = -\sum_j \frac{\alpha_j}{\sqrt{\pi}} \int_0^{\infty} \mathrm{d}t \times \exp\left(-\delta_j t\right) \frac{1}{\sqrt{t-b}} \ln\left(\frac{\sqrt{t-b}+\sqrt{t}}{\sqrt{b}}\right), \tag{23}$$

where

$$b = \left(1 - \mathrm{e}^{-\lambda^2 t}\right)/\lambda^2. \tag{23a}$$

In equation (23) α_j is the e-p coupling constant of the jth branch of LO-phonon modes and defined by [10]

$$\alpha_j = \frac{\left|g_j\right|^2}{4\pi\delta_j^{3/2}}. \tag{24}$$

The polaronic energy of the $n = 0$ Landau level with the zero-phonon state is finally obtained as

$$E_0 = \frac{1}{2}\lambda^2 - \sum_j \frac{\alpha_j}{\sqrt{\pi}} \int_0^{\infty} \mathrm{d}t \exp\left(-\delta_j t\right) \times \frac{1}{\sqrt{t-b}} \ln\left(\frac{\sqrt{t-b}+\sqrt{t}}{\sqrt{b}}\right). \tag{25}$$

Secondly, we calculate the polaronic energies of $n = 1$ states by the IWBPT.

Now we denote the e-p correction $\Delta E_0^{(2)}$ of the $n = 0$ states obtained by PSPT as ΔE_0^{RS}. It is reasonable for calculating the excited states ($n > 0$) to add the e-p energy shift of the ground state to the free Hamiltonian as a reference. Thus the unperturbed Hamiltonian becomes

$$H_0 = H_e + H_{ph} + \Delta E_0^{RS} \tag{26}$$

and the perturbation term is then written as

$$H_1 = H_{e-LO} - \Delta E_0^{RS}. \tag{27}$$

The polaronic energy of the $n = 1$ Landau level then can be calculated by

$$E_1 = E_1^{(0)} + \Delta E_1^{(1)} + \Delta E_1^{(2)}, \tag{28}$$

where

$$E_1^{(0)} = \frac{3}{2}\lambda^2 + \Delta E_0^{RS}$$

and the first-order perturbation correction is obtained as

$$\Delta E_1^{(1)} = -\Delta E_0^{RS}.$$

Thereupon equation (28) is finally becomes

$$E_1 = \frac{3}{2}\lambda^2 + \Delta E_1^{(2)}, \tag{28a}$$

where the second-order perturbation correction to the energy is given by

$$\Delta E_1^{(2)} = \sum_{n=0}^{\infty}\sum_{j\mathbf{k}} \frac{|\langle \mathbf{k}|\langle n, q_z|H_1|1, q_z\rangle|0\rangle|^2}{D_{1n}^{IWBPT}} \tag{29}$$

with

$$D_{1n}^{IWBPT} = \left(E_1^{(0)} + \Delta E_1^{(2)}\right) - \left(E_n^{(0)} + \Delta E_0^{RS} + \delta_j\right). \tag{30}$$

After a straightforward calculation we obtain the following result for the second-order correction to the energy

$$\Delta E_1^{(2)} = -\sum_{j\mathbf{k}} \frac{4\pi\alpha_j\delta_j^{3/2}}{k^2V} \exp\left[-\left(k_\perp/\lambda\right)^2\right] \times \sum_{n=0}^{\infty} \frac{\left(n - k_\perp^2/\lambda^2\right)^2 \left(k_\perp/\lambda\right)^{2n-2}}{n!\left[\left(n+\frac{1}{2}\right)\lambda^2 + \delta_j + k_z^2 - E_1\right]}. \tag{31}$$

4 CR energy and effective mass

The polaronic transition energy from the $n = 0$ to 1 Landau levels, which we call as the CR energy of the polaron in a TMC is given by

$$\hbar\omega_c^* = E_1 - E_0, \tag{32}$$

where ω_c^* is the CR frequency.

The corresponding cyclotron mass is then given by

$$m^* = \frac{eB}{\omega_c^*} = \frac{\hbar\omega_c}{E_1 - E_0} m_b. \tag{33}$$

It is understood that the two states of the system given by an electron in the $n = 0$ Landau level with one LO phonon of frequency ω_c, and only an electron in the $n = 1$ Landau level without phonons, are originally degenerate. A weak e-p coupling can remove the degeneracy and splits the energy level of the system into two. Moreover, the splitting will appears twice at two different magnetic field in a TMC system, since there are two branches of the LO-phonon modes (ω_{+L} and ω_{-L}) interacting with the electrons in this kind of materials, different from binary crystals. We now discuss the splitting of the energy level around the two different LO-frequencies separately. For ease of representation, we introduce the notation λ_j defined by

$$\lambda_j^2 = \delta_j^{-1}\lambda^2 = \omega_c/\omega_{jL}. \tag{34}$$

Here we will focus our attention on the calculations around the magnetic fields of $\lambda_+^2 = 1$ ($\hbar\omega_c = \hbar\omega_{+L}$) and $\lambda_-^2 = 1$ ($\hbar\omega_c = \hbar\omega_{-L}$) respectively, where the splitting may appear.

4.1 About $\lambda_+^2 \approx 1$, namely $\hbar\omega_c \approx \hbar\omega_{+L}$

In this region, the transition energy from $n = 0$ to $n = 1$ Landau levels closes to the LO-phonon energy of the higher-frequency branch in the TMC, so that the contribution of the term corresponding to the higher-frequency branch in the perturbation energy correction (31) becomes dramatically prominent. This situation is then called the resonant e-p coupling. Otherwise, the interaction between the electron and phonons of the lower-frequency branch (ω_{-L}) can be almost neglected. Moreover, one can also found that the dominant term in the sum over n in equation (31) is the term of $n = 0$, which has a smallest energy denominator when λ_+^2 closes to one. Thereupon, a reasonable approximation for the second-order correction of the energy around the level crossing region is

$$\Delta E_1^{(2)} \cong \sum_{\mathbf{k}} \frac{4\pi\alpha_+}{k^2V} \exp\left[-\left(k_\perp/\lambda_+\right)^2\right] \left(k_\perp/\lambda_+\right)^2 \times \frac{1}{E_1 - (E_0+1) - k_z^2}. \tag{35}$$

Furthermore, the condition $E_1 - (E_0 + 1) \ll 1$ is also satisfied in this region, so that the main contribution to the sum over $\mathbf{k}$ comes from the wave-vectors of $k_z = 0$. Thus the equation (35) can be approximately written as

$$\Delta E_1^{(2)} \cong \frac{\sqrt{\pi}}{4} \frac{\alpha_+\lambda_+}{E_1 - E_0 - 1}. \tag{36}$$

The equation for the energy of the $n = 1$ Landau level becomes finally

$$E_1 = \frac{3}{2}\lambda_+^2 + \frac{\sqrt{\pi}}{4} \frac{\alpha_+\lambda_+}{E_1 - E_0 - 1}. \tag{37}$$

Solving equation (37) gives two solutions and the energy level splits into two branches. Hereafter we denote them as $E_{1,u}$ (upper) and $E_{1,d}$ (down) respectively. The transition energy, *i.e.* CR energy of the polaron then can be obtained as follows

$$\hbar\omega_{c,l}^* = E_{1,l} - E_0, \tag{38}$$

where $l = u, d$ stand for, respectively, the "upper" and the "down" energy levels. The polaronic cyclotron masses

also split up into two branches around the magnetic field of $\omega_c = \omega_{+L}$ given by

$$\frac{m_l^*}{m_b} = \frac{\hbar\omega_c}{E_{1,l} - E_0}. \tag{39}$$

To clearly describe the extent of the splitting, we can define a splitting width of CR energy

$$\Delta E_C = \hbar\omega_{c,u}^* - \hbar\omega_{c,d}^*, \tag{40}$$

which will be found sensitive to the e-p coupling strength.

4.2 About $\lambda_-^2 \approx 1$, namely $\hbar\omega_c \approx \hbar\omega_{-L}$

In this region, the resonant e-p coupling appears at the frequency ω_{-L} and only the interaction between the electron and phonons of the lower-frequency branch need to be considered in equation (31). It results in

$$E_1 = \frac{3}{2}\delta_-\lambda_-^2 + \frac{\sqrt{\pi}}{4}\frac{\alpha_-\delta^{3/2}\lambda_-}{E_1 - E_0 - \delta}. \tag{41}$$

The solutions of equation (41) gives another twofold splitting of the polaron CR energy around $\hbar\omega_c = \hbar\omega_{-L}$ and equations (38–40) are also hold.

5 Numerical results and discussion

The numerical computations of the polaronic CR energies and masses as functions of the composition and the magnetic field for several TMC materials are performed. As examples we illustrate the results for the TMCs $Al_xGa_{1-x}As$ and $Ga_xIn_{1-x}As$ in Figures 1–5. The parameters used in the numerical computations are listed in Table 1.

Table 1. Optical phonon energies, dielectric constants, and band-masses of electrons used in the computations. Energy is measured in meV and mass in the electron rest mass.

Materials	$\hbar\omega_{TO}$	$\hbar\omega_{LO}$	ε_0	ε_∞	m_b	a
AlAs [7]	44.88	50.09	10.06	8.16	0.150	5.6611 [16]
GaAs [7]	33.29	36.25	13.18	10.89	0.067	5.6419 [16]
InAs [17]	27.09	30.07	14.61	11.80	0.023	5.6607 [16]

Figures 1 and 2 plot the splitting polaronic cyclotron energies and masses as functions of magnetic field for the materials $Al_xGa_{1-x}As$ and $Ga_xIn_{1-x}As$ with several composition respectively. As was expected, the cyclotron energy and mass are both twofold splitting around the resonant magnetic fields corresponding to $\omega_c \approx \omega_{+L}(\lambda_+^2 \approx 1)$ and $\omega_c \approx \omega_{-L}(\lambda_-^2 \approx 1)$ respectively. One branch in the twofold splitting energy and mass lay above whereas another below the LO-phonon energy and electron band-mass, respectively, at both $\lambda_+^2 = 1$ and $\lambda_-^2 = 1$. It is also found that the two branches of splitting polaronic cyclotron energies and masses increase with increasing the magnetic field. Moreover, the two curves nearly parallel to each other in vicinities of the CR frequencies but separate obviously when the magnetic field parts from the CR value. One of the two curves goes up quickly and linearly with increasing the magnetic field and the other pins to a certain value close to the CR phonon energy. We have chosen the CR magnetic field as two values related to the higher (AlAs-like)- and lower (GaAs-like)-frequency LO phonon energies respectively: $\hbar\omega_+ = 46.4$ meV ($\lambda_+^2 = 1$) and $\hbar\omega_- = 31.14$ meV ($\lambda_-^2 = 1$) for $x = 0.4$, $\hbar\omega_+ = 48.84$ meV ($\lambda_+^2 = 1$) and $\hbar\omega_- = 27.59$ meV ($\lambda_-^2 = 1$) for $x = 0.8$ in our numerical calculation for TMC $Al_xGa_{1-x}As$. The calculated pinning energies are given as around $\hbar\omega_c^* = 44$ meV for $\hbar\omega_+$ and 30 meV for $\hbar\omega_-$ in the case of $x = 0.4$, and around $\hbar\omega_c^* = 44.5$ meV for $\hbar\omega_+$ and 27 meV for $\hbar\omega_-$ in the case of $x = 0.8$ (see Fig. 1a). For $Ga_xIn_{1-x}As$ system, the CR magnetic field has been chosen as two values related to the higher (GaAs-like)- and lower (InAs-like)-frequency LO phonon energies respectively: $\hbar\omega_+ = 34.15$ meV ($\lambda_+^2 = 1$) and $\hbar\omega_- = 21.63$ meV ($\lambda_-^2 = 1$) for $x = 0.47$, $\hbar\omega_+ = 35.78$ meV ($\lambda_+^2 = 1$) and $\hbar\omega_- = 19.7$ meV ($\lambda_-^2 = 1$) for $x = 0.8$ respectively. The results for the pinning energies are given as around

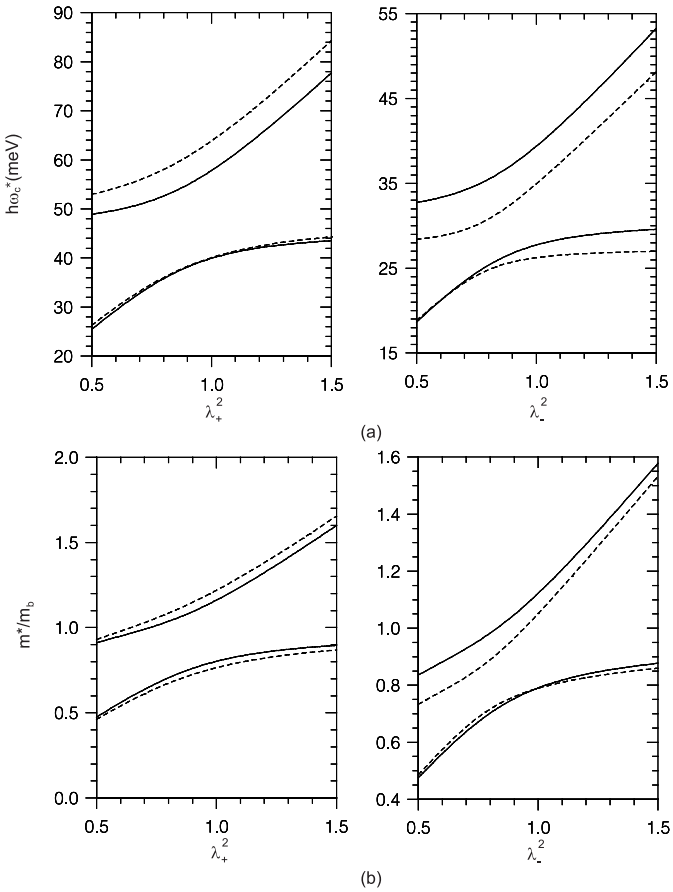

Fig. 1. Polaronic cyclotron energies $\hbar\omega_c^*$ (a) and cyclotron masses m^*/m_b (b) as functions of λ_+^2 and λ_-^2 for the ternary mixed crystal $Al_xGa_{1-x}As$ at composition $x = 0.4$ (solid lines) and $x = 0.8$ (dashed lines) respectively.

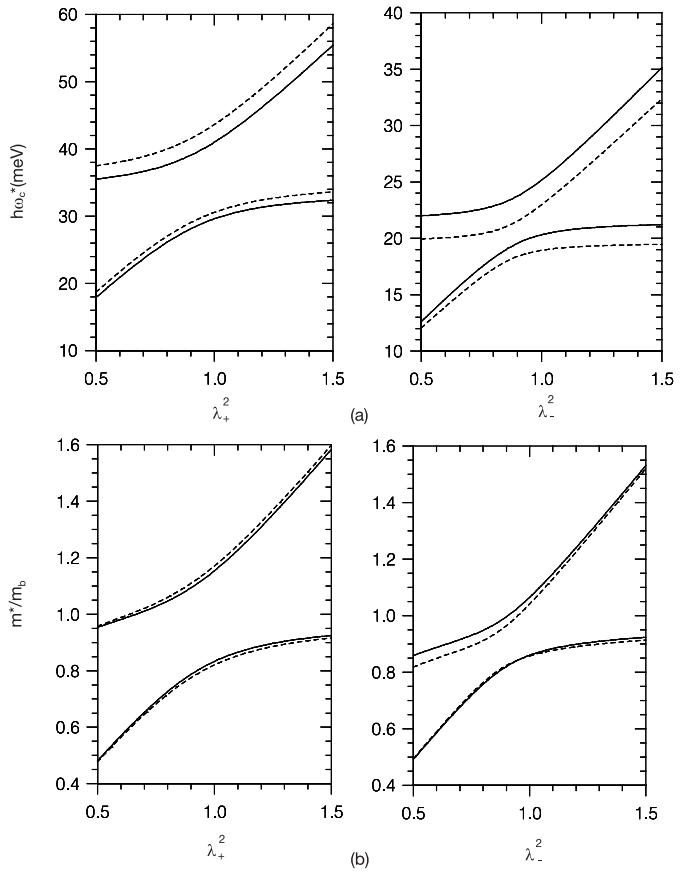

Fig. 2. Polaronic cyclotron energies $\hbar\omega_c^*$ (a) and cyclotron masses m^*/m_b (b) as functions of λ_+^2 and λ_-^2 for the ternary mixed crystal $Ga_xIn_{1-x}As$ at composition $x = 0.47$ (solid lines) and $x = 0.8$ (dashed lines) respectively.

$\hbar\omega_c^* = 32.4$ meV for $\hbar\omega_+$ and 21.2 meV for $\hbar\omega_-$ in the case of $x = 0.47$, and around $\hbar\omega_c^* = 34$ meV for $\hbar\omega_+$ and 19.5 meV for $\hbar\omega_-$ in the case of $x = 0.8$ (see Fig. 2a).

The splitting and pinning of the cyclotron energy and mass around the resonant phonon frequency were observed successfully in binary semiconductors [11,15,16]. The calculated splitting characteristic around the resonance energy for TMC systems here is similar to that shown in the previous experiments for binary systems, but one more splitting of energy and mass is found. Even CR experiments in TMC systems were reported less, the splitting of cyclotron energy and mass have also been observed by the phonon-assisted magneto-tunneling experiments in single and double-barrier heterostructures [18–24]. As expected, two CR peaks have been found experimentally. The splitting and pinning values of the CR energy appear at $44 \sim 48.5$ meV for the AlAs-like LO-phonons and $34.5 \sim 39$ meV for the GaAs-like LO-phonons, respectively, in the $Al_xGa_{1-x}As$ barriers of $x = 0.37, 0.4$ and 0.8. The twofold splitting was also found in $Ga_xIn_{1-x}As$ barrier tunneling at the values lower than the corresponding CR energies. The theoretical results are in agreement with the previous experiments. It is understood that the pinning energy is naturally lower than the corresponding CR energy due to the polaron effects.

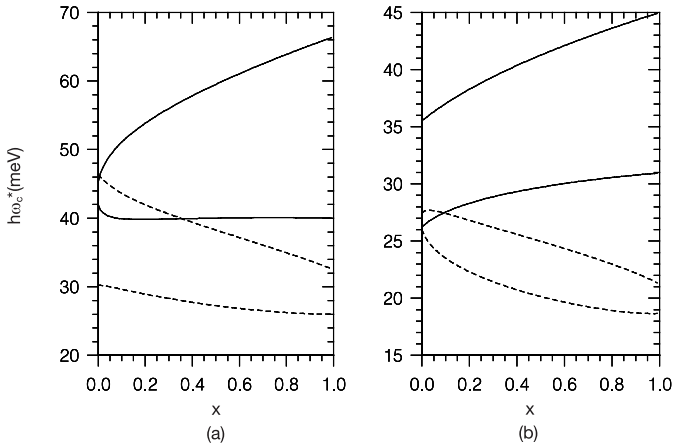

Fig. 3. Polaronic cyclotron energies $\hbar\omega_c^*$ as functions of the composition x at $\lambda_+^2 = 1$ (solid lines) and $\lambda_-^2 = 1$ (dashed lines) for the ternary mixed crystals: (a) $Al_xGa_{1-x}As$, (b) $Ga_xIn_{1-x}As$.

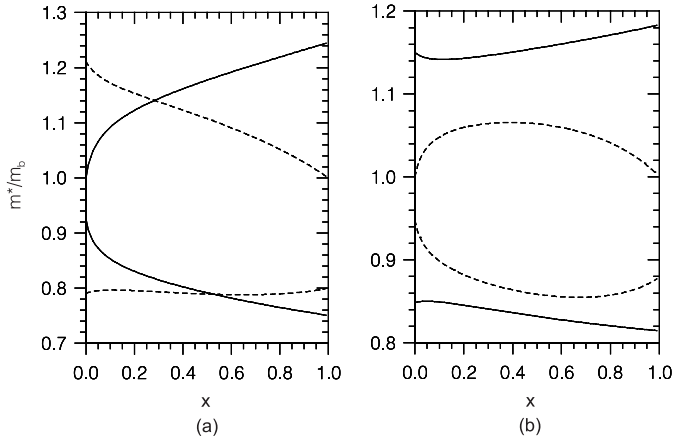

Fig. 4. Polaronic cyclotron masses m^*/m_b as functions of the composition x at $\lambda_+^2 = 1$ (solid lines) and $\lambda_-^2 = 1$ (dashed lines) for the ternary mixed crystals: (a) $Al_xGa_{1-x}As$, (b) $Ga_xIn_{1-x}As$.

To understand the influences of the composition x on the polaronic cyclotron energies, cyclotron masses and splitting widths of cyclotron energy, we have also plotted the curves of the cyclotron energies $\hbar\omega_c^*$, relative cyclotron masses m^*/m_b and splitting widths of cyclotron energy ΔE_C as functions of the composition x at $\lambda_+^2 = 1$ and $\lambda_-^2 = 1$ for the materials $Al_xGa_{1-x}As$ and $Ga_xIn_{1-x}As$ in Figures 3–5, respectively. The non-linear variations of the polaronic cyclotron energies and masses with the composition x are found for both the TMC $Al_xGa_{1-x}As$ and $Ga_xIn_{1-x}As$. The non-linearity is easily understood by the non-linear change of the e-p coupling with the composition [10]. The phenomena can be explained as that the two polarization-waves due to the A-C and B-C ion-pairs disturb by each other.

In summary, we have studied the cyclotron resonance of polarons in ternary mixed crystals by using Rayleigh-Schrödinger perturbation theory and improved Wigner-Brillouin perturbation theory. The numerical

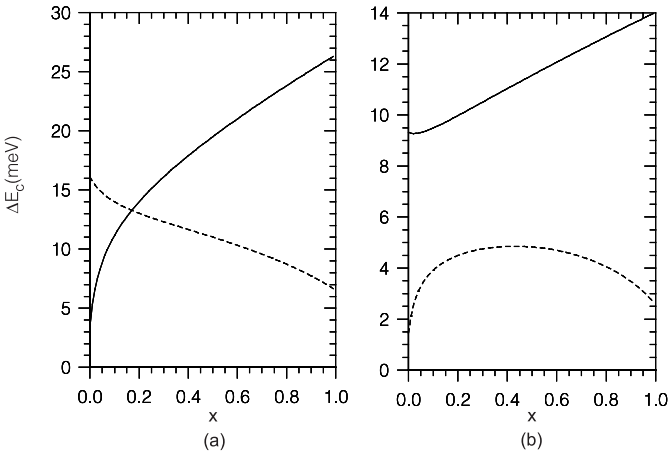

Fig. 5. Splitting widths of cyclotron energy ΔE_C as functions of the composition x at $\lambda_+^2 = 1$ (solid line) and $\lambda_-^2 = 1$ (dashed line) for the ternary mixed crystals: (a) $Al_xGa_{1-x}As$, (b) $Ga_xIn_{1-x}As$.

results for $Al_xGa_{1-x}As$ and $Ga_xIn_{1-x}As$ show that the polaronic cyclotron energy and mass successively splits twice due to the coupling of the electron with two branches of longitudinal optical phonon modes. A non-linear dependence of the phonon contributions to the polaronic cyclotron energy and mass on the composition x is found.

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